

Fun with Mathematics:

Do you know that all the cubic numbers can be expressed as difference of two square numbers? If it is so, how to prove it?

Let a^3 is a cubic number where a is any positive integer.

Let us now assume $a^3 = x^2 - y^2$
Or $a^2 \cdot a = (x+y)(x-y)$

From this equation, we can equate
 $x+y = a^2$ -----(1)
 $x-y = a$ -----(2)

Now adding eqn. (1) and (2), we get
 $2x = a^2 + a = a(a+1)$
 $x = a(a+1)/2$ -----(3)

Similarly, subtracting (2) from (1), we get
 $y = a(a-1)/2$ -----(4)

Eqn. (3) and (4) determine value X and Y wrt. a. Now let us verify it with numerical examples.

a	x	y	Example	Remarks
0	0	0	$0^3 = 0 = 0^2 - 0^2 = 0 - 0$	
1	1	0	$1^3 = 1 = 1^2 - 0^2 = 1 - 0$	The formula $a^3 = x^2 - y^2$ is valid.
2	3	1	$2^3 = 8 = 3^2 - 1^2 = 9 - 1$	
3	6	3	$3^3 = 27 = 6^2 - 3^2 = 36 - 9$	
4	10	6	$4^3 = 64 = 10^2 - 6^2 = 100 - 36$	
5	15	10	$5^3 = 125 = 15^2 - 10^2 = 225 - 100$	
6	21	15	$6^3 = 216 = 21^2 - 15^2 = 441 - 225$	
7	28	21	$7^3 = 343 = 28^2 - 21^2 = 784 - 441$	
8	36	28	$8^3 = 512 = 36^2 - 28^2 = 1296 - 784$	
9	45	36	$9^3 = 729 = 45^2 - 36^2 = 2025 - 1296$	
10	55	45	$10^3 = 1000 = 55^2 - 45^2 = 3025 - 2025$	

and so on.

The above true table shows certain special features. When the value of a is gradually increased with the increment of 1, the value of x is also increases, and the increments are in AP. The value of y is also increases accordingly as shown in the table. You can see there that the same number as that of the values of x are appearing there but with a shift of one house. Is not really interesting?

(To be continued)

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